

Maths Summer Task: Algebraic Manipulation

$$\begin{array}{l} (\sqrt[3]{32})^3 \\ x^5 \\ 3^{-4} = \frac{1}{3^4} \\ 25^{1/2} \\ y^3 \times y^6 = y^9 \\ \sqrt[3]{27} \end{array}$$

Key Skills

Problem solving
Independent work

Your Task:

These are 10 non-calculator problems.

The emphasis is on you to show logical, clear thinking and presentation of solutions. Someone else is reading your work and you have to allow them to follow your thought processes.

Tenacity is important; don't give up if at first you don't succeed.

Show a clear mathematical explanation for each solution.

1)

What number, when multiplied by itself, is equal to 27×147 ?

2)

What is the smallest number divisible by 1, 2, 3, 4, 5, 6, 7, 8 and 9?

3)

What is the units digit of 703^{703} ?

4)

Show that, if x is a real number then $2x^2 + 6x + 9$ is always positive.

5)

Find all real solutions of the equation

$$(x^2 - 7x + 11)^{(x^2 - 11x + 30)} = 1$$

6)

Eight points are placed at the vertices of a cube, side length = 2cm. How many different triangles can be made by joining three of the points? What is the area of the smallest of these triangles?

7)

The number

$$(10^2 - 8^2)(9^2 - 7^2) \times \dots \times (4^2 - 2^2)(3^2 - 1^2)$$

can be written in the form $k \times 2^n$, where k and n are positive integers and k is odd. What are the values of k and n ?

	<p>8)</p> <p>Five numbers are arranged in order from least to greatest:</p> x, x^3, x^4, x^2, x^0 <p>Where does $-x^{-1}$ belong in the list above?</p> <p>9)</p> <p>What is the area of the triangle whose vertices lie on the points</p> $(2,3), (-1, -2), (-9,10) ?$ <p>10)</p> <p>What is the equation of the set of points that are equidistant from (2, 4) and (1, 1)?</p>
Your starting point:	<p>This is a challenge and the problems will not be easy to solve. You must spend time and effort doing this work.</p> <p>A strong solution is one in which your logic is clear and working is well-presented; the solution is complete.</p> <p>Write up your results on A4 lined paper. You should be able to present several pages of neat, well-organised notes, diagrams and calculations with conclusions.</p>
How you will be assessed:	<p>Grade A – Your work: Solutions are complete, or mainly complete. Working and explanations are clear and either completely or mainly precise and exact. The work shows mathematical imagination and skill, and may well show evidence of independent study or research. The work is mathematically accurate.</p> <p>Grade C – Your work: Solutions are sometimes or partially complete. There is evidence of application and correct methods, but the work also reveals some incorrect methods, calculations or ideas. Work is presented and argued in a satisfactory way.</p> <p>Grade E – Your work: E grade: work is not complete. There is evidence of significant misconceptions, or the work is badly presented or poorly argued.</p>