Fleet Primary School
Calculation Policy

Our aims are to ensure consistency and progression in maths teaching, and enjoyment and achievement in maths learning throughout the whole school.

How will we help children to develop their understanding of calculation?

Children are introduced to the processes of calculation through practical, visual, oral and mental activities. As children begin to understand the underlying ideas they develop ways of visualising and recording to support their thinking and calculation methods. They learn to interpret and use the signs and symbols involved, choosing when to use which methods.

Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children’s mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally.

By the end of Year 6 children are equipped with mental and written methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

It is important to remember that written methods do not replace mental methods of calculation.

How to achieve consistency

It is imperative that...

- There is a consistent set of images and models which are developed throughout the children's primary years and which enable children to access and transfer their learning, eg.

  beaded number line ------ marked number line ------- empty number line

- The vocabulary used to describe methods should be agreed by each teacher so the words used are the same from one year to the next to avoid confusion, and these should be shared with support staff and parents

- There is agreement as to the development of written methods of calculation, to avoid abrupt (and to the children, unexplained) changes in which are taught.

When children leave Fleet Primary they should:

- have a secure knowledge of number facts and a good understanding of the four operations;
be able to use this knowledge and understanding to estimate their answer and carry out mental calculations involving one-digit and two-digit numbers;

make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;

have an efficient, reliable, compact written method of calculation for each operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;

be able to solve a variety of problems by identifying the relationship between numbers involved, identifying the operation(s) needed and selecting and using a suitable calculation method.

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another. The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers - ‘number sense’ or a ‘feel’ for numbers - is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

Secure mental calculation requires the ability to:

- recall key number facts instantly;
- use taught strategies to work out the calculation;
- understand how the rules and laws of arithmetic are used and applied.
Expectations for Number facts

Year 1
- Derive and recall all pairs of numbers with a total of 10 and addition facts for totals to at least 5; work out the corresponding subtraction facts
- Count on or back in ones, twos, fives and tens and use this knowledge to derive the multiples of 2, 5 and 10 to the tenth multiple
- Recall the doubles of all numbers to at least 10

Year 2
- Derive and recall all addition and subtraction facts for each number to at least 10, all pairs with totals to 20 and all pairs of multiples of 10 with totals up to 100
- Understand that halving is the inverse of doubling and derive and recall doubles of all numbers to 20, and the corresponding halves
- Derive and recall multiplication facts for the 2, 5 and 10 times-tables and the related division facts; recognise multiples of 2, 5 and 10

Year 3
- Derive and recall all addition and subtraction facts for each number to 20, sums and differences of multiples of 10 and number pairs that total 100
- Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and the corresponding division facts; recognise multiples of 2, 5 or 10 up to 1000

Year 4
- Use knowledge of addition and subtraction facts and place value to derive sums and differences of pairs of multiples of 10, 100 or 1000
- Identify the doubles of two-digit numbers; use these to calculate doubles of multiples of 10 and 100 and derive the corresponding halves
- Derive and recall multiplication facts up to 12 × 12, the corresponding division facts and multiples of numbers to 10 up to the tenth multiple
- Identify pairs of fractions that total 1

Year 5
- Use knowledge of place value and addition and subtraction of two-digit numbers to derive sums and differences and doubles and halves of decimals (e.g. 6.5 ± 2.7, half of 5.6, double 0.34)
- Recall quickly multiplication facts up to 12 × 12 and use them to multiply pairs of multiples of 10 and 100; derive quickly corresponding division facts.
- Identify pairs of factors of two-digit whole numbers and find common multiples (e.g. for 6 and 9)

Year 6
- Use knowledge of place value and multiplication facts to 12 × 12 to derive related multiplication and division facts involving decimals (e.g. 0.8 × 7, 4.8 ÷ 6)
- Use knowledge of multiplication facts to derive quickly squares of numbers to 12 × 12 and the corresponding squares of multiples of 10
- Recognise that prime numbers have only two factors and identify prime numbers less than 100; find the prime factors of two-digit numbers

Application of number bonds – calculate don’t count. (22+7 = 20 + (2+7))
Strategies for teaching number facts:
N.B. Number facts should be explicitly taught, making connections with prior knowledge and investigating patterns to help make sense of these facts. Fundamental to all these calculations is the understanding of Place Value: that the position of a digit determines its value.

- Speed tests *Speed tests should be done in this order and repeated regularly:*
- 2, 10, 5, 4, 3, 6, 9, 8 and 7
- Roll the die & multiply
- Times table/ number bond tournament (all children on the list going one on one to move up or down the league table each day)
- Chanting
- Ping pong
- Using a 100 square or counting stick to rehearse and order multiples and spot patterns
- Factor bugs (finding factors of a number drawn as legs on a bug)
- Number properties guessing games
- Snap games
- Number tracks – landmarked, empty, different orientation

Written methods of calculation

The 1999 Numeracy Framework set out progression in written methods of calculation that highlighted how children would move from informal methods of recording to expanded methods that are staging posts to a compact written method for each of the four operations. This policy explains the methods to be taught at Fleet, based on this progression.

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads. We want children to know that they have a reliable, written method to which they can turn when the need arises.

Progression through Calculations

Up to Year 3, the emphasis should be on children working mentally. Once written methods are introduced, mental skills must be kept sharp by continuing to develop and apply them with appropriate examples.

When do children need to start recording?
It is important to encourage children to look first at the problem and then get them to decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording such as formal written methods.

<table>
<thead>
<tr>
<th>Reception</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making a record of a calculation</td>
<td>Jotting to support a new mental strategy (but ditching jotting when secure in that strategy)</td>
<td>Explaining a mental strategy</td>
<td>Developing written methods</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
What about children at different stages of attainment?
In many classes, children will be at different stages in their move towards efficiency. This process should not be rushed; children should be moved on when they are ready. That is why it is so crucial for us to be familiar with all stages of calculation.

How can children’s readiness for written calculations be judged?
Judgements will need to be made as to whether pupils possess sufficient of these skills to progress. Different prerequisite skills are needed for each operation.

A short list of criteria for readiness for written methods of addition and subtraction would include:
- Do children know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and ones?
- Do they use and apply the commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

A short list of criteria for readiness for written methods for multiplication and division are:
- Do the children know the 2, 3, 4, 5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand place value?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2-digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don’t know?
- Can they explain their mental strategies orally and record them using informal jottings?

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

By the end of year 6, children will have a range of calculation methods, mental and written. Selection of method will depend upon the numbers involved.

Children should be encouraged to
- consider if a mental calculation would be appropriate before using written methods.
- approximate their answers before calculating.
- check their answers after calculation using an appropriate strategy.
CALCULATION PROGRESSION FOR ADDITION

<table>
<thead>
<tr>
<th>Sum</th>
<th>Plus</th>
<th>Altogether</th>
<th>Total</th>
</tr>
</thead>
</table>

Mental Calculation Strategies for Addition
These are a selection of mental calculation strategies which should continue to be practiced and used in lessons right up to year 6.

Mental recall of number bonds

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6 + 4</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>25 + 75</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

Using number bonds to add many numbers
5 + 2 + 8 + 9 + 1; combine 9 and 1, 8 and 2 to see the answer is 25

Use near doubles
6 + 7 = double 6 + 1 = 13

Addition using partitioning and recombinining
34 + 45 = (30 + 40) + (4 + 5) = 79

Counting on or back in repeated steps of 1, 10, 100, 1000
86 + 57 = 143 (by counting on in tens and then in ones)
460 - 300 = 160 (by counting back in hundreds)

Add the nearest multiple of 10, 100 and 1000 and adjust
24 + 19 = 24 + 20 − 1 = 43
458 + 71 = 458 + 70 + 1 = 529

Use the relationship between addition and subtraction
36 + 19 = 55
55 − 19 = 36
19 + 36 = 55
55 − 36 = 19

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

See NNS Framework Section 5, pages 30-41 and Section 6, pages 40-47
Addition – Written Calculation, Models & Images

Nursery & Reception

Children are taught to recognise numerals 1-10 (Nursery) 1-20 (Reception). They learn that number names are numerals.

Children count from 1-10, understanding that 0 is ‘nothing’, finding one more or one less than a number from 1 to 10.

They know that \( * * * = 3 \), moving or pointing to objects counted as they count. Emphasising the last number (cardinal number) to show that they count the objects and the last one is how many there are. If you move objects around, there are still 3. If you spread them out, there are still 3.

Emphasise \( = \) as a symbol of balance or equivalence or ‘is the same as’. This will be the first symbol that children learn.

Follow this with using the inequality symbols to compare two quantities.

Children are able to give you a number of objects from a bigger group eg. give you 8 from a pile of 20.

They can count with help to 20, observing number relationships and patterns in the environment.

They are able to combine two groups of objects to make a given total of objects

**Some teaching strategies include:** using flash cards at carpet times, lining up numbers, using display to support recognition of numerals, counting incorporated into play/practical situations, using counting songs, using **Numicon** which children to visualise number, add and subtract. It is also useful for number bonds. It is not about counting, but having a mental image of a number so children know instantly what 2 or 5 looks like.
Reception & Year 1

Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

Begin with a number track

![Number Track](image)

Play games to encourage adding two in one leap as well as one.

They use landmarked number lines, number squares and other practical resources to support calculation and teachers *demonstrate* the use of the number line to represent a 'number sentence'.

\[
3 + 2 = 5
\]

---

\[
8 + 5 = 13
\]

Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones. Some children may be able to count on in 2s. Encourage using number facts to add bigger jumps rather than always counting on in ones.

Encourage and teach these strategies so that children are calculating not just counting:

- Knowing and using the stories of numbers i.e. 6 is 5 + 1, 4 + 2 etc – use cuisinaire, tens frames, cherry diagrams and bar models to support this
- Bridge 10 by splitting numbers into parts (tens frames support this) i.e \(8 + 4 = 8 + 2 + 2\)
- Using knowledge of place value i.e. \(20 + 2 = 22\) and \(22 + 3 = 25\) (using \(2 + 3 = 5\))
The 100 square should be introduced by building it out of number tracks.

Explore it by filling in empty 100 squares, or ones with parts missing etc.

The 100 square should be introduced with children using it to count on in 1s and to count forward or back in multiples of 2, 5 and 10.

Dienes apparatus, as well as tens frames, Cuisenaire and Numicon, should be used to show the first stages of partitioning eg. 23 is 20 and 3 more

Children will be shown ‘spider counting’ using a 100 square eg. 26 + 30 (three tens) = 56. The spider can only move up or down in tens. This can then be used to help add multiples of ten and adjust. 200 squares should also be shown to familiarise children with going beyond 100.

Children will begin to use ‘empty number lines’ as an alternative image themselves starting with the larger number and counting on.
**Y2**

Continue spider counting and use the hundred square/200 hundred square or a landmarked number line to introduce adding nearly numbers (adjusting) using 9 and 11.

✓ First counting on in tens and ones.

\[ 34 + 23 = 57 \]

✓ Then helping children to become more efficient by adding the ones in one jump (by using the known fact 4 + 3 = 7).

\[ 34 + 23 = 57 \]

✓ Followed by adding the tens in one jump and the ones in one jump.

\[ 34 + 23 = 57 \]

✓ Bridging through ten can help children become more efficient.

\[ 37 + 15 = 52 \]

Children will continue to ‘build the number’ using dienes apparatus, tens frames and Numicon.

**Bring in place value counters as soon as possible.**

This will allow them to begin to refer to ones, tens and hundreds.
Explore **whole/part** relationships using bar modelling to explore fact families...

![Bar model](image)

Use the image for understanding number relationships at this stage rather than a strategy for calculation.

Identify and explore the inverse relationships here. Introduce empty box calculations e.g. \(8 + \square = 10\). \(10 - \square = 4\). Reinforce ‘=’ meaning ‘the same as’.

**Whole/Part relationship to be referred back to throughout subsequent years.**

Children should know that these words mean addition: add, sum, plus, altogether and total.

✓ Count on from the largest number irrespective of the order of the calculation.

\[38 + 86 = 124\]

![Count on from the largest number](image)

✓ Compensation

\[49 + 73 = 122\]

![Compensation](image)
Children will be taught how to partition numbers, using dienes apparatus to support their understanding. They should know that $124 = 100 + 20 + 4$.

They will use place value cards and counters to support partitioning. Some children will begin to use place value to partition, add and recombine.

**Y3**

$$123 + 234 =$$

100 + 200 = 300

20 + 30 = 50

3 + 4 = 7 so the whole answer is **357**

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Move on to begin expanded columnar addition mirrored with dienes and **place value counters** to explain the carrying stage.
Split screen to show compact alongside expanded method after introductory lesson(s). Encourage children to move on to compact as soon as secure.

Continue to teach these mental strategies:
- Adding near doubles
- Adding nearly numbers
- Making bonds for shortcuts if adding a pile of numbers

**Y4**

From this, children will continue to carry below the line.

\[
\begin{array}{ccc}
625 & + & 48 \\
783 & + & 42 \\
367 & + & 85 \\
673 & + & 825 & + & 452 \\
& 1 & & & 11
\end{array}
\]

Using the same method, children will:
- add several numbers with different numbers of digits;
- begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.

**Yrs 5 & 6**

nb. Children should use mental methods if the numbers lend themselves.

nb. Piles of numbers – extend through piles of numbers rather than bigger numbers, or vary the number of decimal places.
CALCULATION PROGRESSION FOR SUBTRACTION

<table>
<thead>
<tr>
<th>Minus</th>
<th>Take-away</th>
<th>Less</th>
<th>Reduce</th>
<th>Find the difference</th>
</tr>
</thead>
</table>

**Mental Calculation Strategies for Subtraction**

These are a selection of mental calculation strategies:

- **Mental recall of addition and subtraction facts**
  - 10 - 6 = 4
  - 20 - 17 = 3
  - 17 - □ = 11
  - 10 - □ = 2

- **Find a small difference by counting up**
  - 82 – 79 = 3

- **Counting on or back in repeated steps of 1, 10, 100, 1000**
  - 86 - 52 = 34 (by counting back in tens and then in ones)
  - 460 - 300 = 160 (by counting back in hundreds)

- **Subtract the nearest multiple of 10, 100 and 1000 and adjust**
  - 24 - 19 = 24 - 20 + 1 = 5
  - 458 - 71 = 458 - 70 - 1 = 387

- **Use the relationship between addition and subtraction**
  - 36 + 19 = 55
  - 55 – 19 = 36
  - 19 + 36 = 55
  - 55 – 36 = 19

*MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.*
Subtraction – Written Calculation, Models & Images

Nursery & Reception
Children are able to understand the concept of taking away from a group of objects and that there will be ‘fewer’ objects. They will talk about ‘less/more’ than.

Children will join in with counting back in 1s – not just from 10! - eg. rocket launch and maths songs

Reception & Y1
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures and beginning to use a number track.

They use numberlines and practical resources to support calculation. Teachers demonstrate the use of the numberline.

6 – 3 = 3

The numberline should also be used to show that 6 - 3 means the ‘difference between 6 and 3’ or ‘the difference between 3 and 6’ and how many jumps they are apart.
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

\[ 13 - 5 = 8 \]

Use Numicon or Cuisenaire to introduce the whole-part model in order to use number facts to solve subtraction.

Begin to calculate using number facts where there is no bridging eg 25-4 =
Then begin to bridge using known facts eg 25 – 6 = 25 – 5 - 1

**Y2**

Children will begin to use empty number lines to support calculations.

**Counting back – ‘rob the bank’**.
Children will remember the concept of robbing the bank problems as the robber steals in 1s and 10s. When we’ve found out how much is left, we can catch the robber and say ‘take him away’!

✓ First counting back in tens and ones.

\[ 47 - 23 = 24 \]

✓ Then helping children to become more efficient by subtracting the ones in one jump (by using the known fact 7 – 3 = 4).
\[ 47 - 23 = 24 \]

-3  -10  -10

\[ 24 \quad 27 \quad 37 \quad 47 \]

✓ Subtracting the tens in one jump and the ones in one jump.

\[ 47 - 23 = 24 \]

-3  -20

\[ 24 \quad 27 \quad 47 \]

✓ Bridging through ten can help children become more efficient.

\[ 42 - 25 = 17 \]

-3  -2  -20

\[ 17 \quad 20 \quad 22 \quad 42 \]
Counting on – ‘bald-headed man’

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on. The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with ‘taking away’.

**Example:**

\[45 - 23 = \]

1) I draw a number line with the big number at the end and 'take away' 23

2) I find the next comfy cafe number after 23 where I can stop and have a rest. It's the next multiple of 10 which is 30!

3) I find the cafe number (tens number ending in 0) before 45. It's 40.

4) I write down how far each jump is then add them up. This is my answer!

5) Now I can draw in my bald man face!

\[+7 \quad +10 \quad +5\]

I know that \(7 + 10 + 5 = 22\) so that answer is 22!

N.B. We call multiples of ten here ‘cafe numbers’ where we can have a rest before counting the next jump. It is important to continue to call them multiples of ten as well, showing children how they can use number bonds to ten, to find out how big their jump is.

Link to bar model – whole/part model to show relationship between numbers:
Some children may bridge ten (going from 47 to 57 rather than to 50). This is fine too:

Count up from 47 to 82 in jumps of 10 and jumps of 1.

\[ 82 - 47 \]

Help children to become more efficient with counting on by:

- Subtracting the ones in one jump;
- Subtracting the tens in one jump and the ones in one jump;
- Bridging through ten.

**Y3**

Children will continue to use empty number lines – the **bald headed man method** - with increasingly large numbers.

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

**Partitioning and decomposition**

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

**NOTE:** When solving the calculation 89 – 57, children should know that 57 does **NOT EXIST AS AN AMOUNT** - it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

\[
89 \quad = \quad 80 \quad \text{and} \quad 9 \\
- \quad 57 \quad = \quad 50 \quad \text{and} \quad 7 \\
\_ \quad = \quad 30 \quad \text{and} \quad 2 = 32
\]

*Initially, the children will be taught using examples that do not need the children to exchange.*

Split screen with dienes first then place value counters, then compact.
Next the children will begin to exchange. Most children in year 3 will only just be ready for this so tread carefully. It is essential that children understand how to use a number line and have a secure understanding of place value before progressing to this stage. There are lots of pitfalls they can fall into if they don’t have the right foundations.

\[
\begin{array}{c}
71 \\
\hline
- 46 \\
\hline
\end{array}
\]

| Step 1 | \begin{align*}
70 & \text{ and } 1 \\
40 & \text{ and } 6 \\
\hline
\end{align*} |
|---|---|
| Step 2 | \begin{align*}
60 & \text{ and } 11 \\
40 & \text{ and } 6 \\
20 & \text{ and } 5 = 25 \\
\hline
\end{align*} |

This would be recorded by the children as

\[
\begin{array}{c}
60 \\
\hline
1 \\
\hline
- 40 \\
\hline
6 \\
\hline
20 \\
\hline
5 = 25 \\
\hline
\end{array}
\]

The calculation should be read as e.g. take 6 from 1. Get the whole class to chant ‘you can’t do it’ and make sure they don’t come up with an answer of 5!

Tell the children there are plenty of ones hanging around in the tens column, grouped as tens. We exchange one ten for ten ones and put them in the ones column. Spin around and come back to it. Now we can take 6 from 11!

The National Library of Virtual Manipulatives (nvlm.usu.edu) has a useful interactive to demonstrate the concept of exchanging 1 ten for 10 ones. Children should know that ones line up under ones, tens under tens, and so on.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used. This method can continue to be practised for children not ready for decomposition.

\[102 - 89 = 13\]
Y4

Partitioning and decomposition

\[
754 = \\
- 86
\]

Step 1

\[
\begin{array}{c}
700 \\
- 80 \ \text{and} \ 6
\end{array}
\]

Step 2

\[
\begin{array}{c}
700 \ \text{and} \ 40 \ \text{and} \ 14 \ \text{(adjust from T to U)} \\
- 80 \ \text{and} \ 6
\end{array}
\]

Step 3

\[
\begin{array}{c}
600 \ \text{and} \ 140 \ \text{and} \ 14 \ \text{(adjust from H to T)} \\
- 80 \ \text{and} \ 6
\end{array}
\]

\[
600 \ \text{and} \ 60 \ \text{and} \ 8 = 668
\]

This would be recorded by the children as

\[
\begin{array}{c}
600 \\
700 \\
- 80 \ \text{and} \ 6
\end{array}
\]

\[
\begin{array}{c}
140 \\
\text{and} \ 14
\end{array}
\]

\[
600 \ \text{and} \ 60 \ \text{and} \ 8 = 668
\]

Decomposition

\[
\begin{array}{c}
614 \\
754 \\
- 86
\end{array}
\]

\[
\begin{array}{c}
668
\end{array}
\]

Children should:

✓ be able to subtract numbers with different numbers of digits using place value knowledge;
✓ using this method, children should also begin to find the difference between two three-digit sums of money, with or without ‘adjustment’ from the pence to the pounds;
✓ know that decimal points should line up under each other.

For example:

\[
\begin{array}{c}
£8.95 = 8 \ \text{and} \ 0.9 \ \text{and} \ 0.05 \\
- £4.38 = 4 \ \text{and} \ 0.3 \ \text{and} \ 0.08
\end{array}
\]

\[
\begin{array}{c}
1
\end{array}
\]

\[
\begin{array}{c}
8 \ \text{and} \ 0.8 \ \text{and} \ 0.15 \ \text{(adjust from T to U)} \\
8.85
\end{array}
\]
Alternatively, children can set the amounts to whole numbers, i.e. 895p – 438p and convert to pounds after the calculation.

**NB If your children have reached the concise stage they will then continue this method through into years 5 and 6. See year 5 for what to do if there is a 0 in your larger number.**

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

511 – 197 = 314

### Year 4 and beyond

**Partitioning and decomposition**

Step 1

\[
\begin{align*}
754 & = 700 \text{ and } 50 \text{ and } 4 \\
& - 286 \text{ (adjust from T to U)} \\
& - 200 \text{ and } 80 \text{ and } 6
\end{align*}
\]

Step 2

\[
\begin{align*}
700 \text{ and } 40 \text{ and } 14 & \text{ (adjust from T to U)} \\
& - 200 \text{ and } 80 \text{ and } 6
\end{align*}
\]

Step 3

\[
\begin{align*}
600 \text{ and } 140 \text{ and } 14 & \text{ (adjust from H to T)} \\
& - 200 \text{ and } 80 \text{ and } 6 \\
400 \text{ and } 60 \text{ and } 8 & = 468
\end{align*}
\]

This would be recorded by the children as

\[
\begin{align*}
600 \text{ and } 140 & \\
700 \text{ and } 50 \text{ and } 14 & \\
& - 200 \text{ and } 80 \text{ and } 6 \\
400 \text{ and } 60 \text{ and } 8 & = 468
\end{align*}
\]
Children able to decomposition confidently can then move on to learning what happens if there is a 0 in your larger number:

1) 3 - 6... you can’t do it! So you look at the tens but there’s a 0 there. So you go to the hundreds.

2) The 3 becomes a 2
The 0 becomes a 10 BUT then you take 1 of these tens away so there are 9 tens left in the tens column in the end. Now there is 13 in the units column.

3) Turn around and breathe! Now we can begin solving the problem

Does my answer look sensible?
I can check it by adding the answer to the small number and see if I come out with the top one!
This is using the inverse to check my maths!

Children should
✓ be able to subtract numbers with different numbers of digits using place value knowledge;
✓ begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
✓ know that decimal points should line up under each other.

N.B. If your children have reached the concise stage they will then continue this method through into year 6. They will not go back to using the expanded methods.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.
Children do not need to continue using the 0 but can start the number line from their smaller number.

£1.32 - 92p = 40p
Decomposition

Children should:
- be able to subtract numbers with different numbers of digits;
- be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;
- know that decimal points should line up under each other.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

3002 – 1997 = 1005
CALCULATION PROGRESSION
FOR MULTIPLICATION

<table>
<thead>
<tr>
<th>Times</th>
<th>Lots of...</th>
<th>Product</th>
<th>Double</th>
<th>Groups of...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple</td>
<td>Repeated addition</td>
<td>Array</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Mental Calculation Strategies for Multiplication**

These are a selection of mental calculation strategies:

- **Doubling and halving – this is really important!**
  Children need to know doubles from 1 – 9 by heart and related halves. They must also be taught how to relate these to doubling multiples of 10 and then to partition and double 2- and 3-digit numbers.

- **Applying the knowledge of doubles and halves to known facts e.g. 8 x 4 is double 4 x 4**

- **Using multiplication facts**
  - Tables should be taught everyday from the last term of year 1 onwards, either as part of the mental oral starter or other times as appropriate within the day, particularly using the times table tournament and chanting through transitions.

<table>
<thead>
<tr>
<th>Year</th>
<th>Multiplication Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>2 x table, 10 x table, 5 x table</td>
</tr>
<tr>
<td>3</td>
<td>2 x table, 10 x table, 5 x table, 4 x table, 3 x table, 6 x table</td>
</tr>
<tr>
<td>4</td>
<td>Derive and recall all multiplication facts up to 12 x 12</td>
</tr>
<tr>
<td>5 &amp; 6</td>
<td>Derive and recall quickly all multiplication facts up to 12 x 12</td>
</tr>
</tbody>
</table>

- **Using and applying multiplication facts to scaled related numbers**
  Children should be able to utilise their tables knowledge to derive other facts.
  - e.g. If I know 3 x 7 = 21, what else do I know?
    30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc

- **Use closely related facts already known**
  - 13 x 11 = (13 x 10) + (13 x 1)
    = 130 + 13
    = 143

- **Multiply by 10 or 100**
Know that the effect of multiplying by 10 is a shift in the digits one place to the left. Know that the effect of multiplying by 100 is a shift in the digits two places to the left.

- **Partitioning and recombining**
  
  \[ 23 \times 4 = (20 \times 4) + (3 \times 4) = 80 + 12 = 92 \]

- **Use of factors**

  *Children should find prime factors and know prime and composite numbers.*
  *They should understand and identify shared factors.*

  e.g. \( 8 \times 15 = 8 \times 5 \times 3 \)

---

**Multiplication – Written Calculation, Models & Images**

**Nursery & Reception**

Children will experience equal groups of objects, creating patterns and sorting objects.

**Reception & Y1**

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem-solving activities involving equal sets or groups.

Children will count on in 2s, 5s and 10s use ‘finger multiplication’ – each finger today is worth 2. How many if I hold up 4 fingers?

Children will develop their understanding of multiplication and use jottings to support calculation:

- **Repeated addition – model with Cuisenaire and Numicon**

  3 times 5 is \( 5 + 5 + 5 = 15 \) or 3 lots of 5 or \( 5 \times 3 \)

  Repeated addition can be shown easily on a number line:

  \[
  5 \times 3 = 5 + 5 + 5
  \]
Bring in bar model to show repeated addition.

![Bar Model](image)

**Y2**

Children will continue to use:

- **Repeated addition**
- **Commutativity**

Children should know that $3 \times 5$ has the same answer as $5 \times 3$. This can also be shown on the number line:

![Number Line](image)

- **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

```
  O O O O O O
  O O O O O O  5 x 3 = 15
  O O O O O O
```

$5 \times 3 = 15$

Continue to split screen with bar model.
Children will also learn how to double (0-10) and halve (0-20) using objects to support initial understanding.

**Y3**

Children will continue to use:

- **Repeated addition**

4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

Children should use number lines or bead bars to support their understanding.

![Number Line](image)

**Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

![Array](image)

Children will also develop an understanding of

- **Scaling – use cuisinaire and bar model**

e.g. The red ribbon is 4 times as long as the blue ribbon. How long is the red ribbon?

[Blue ribbon: 5 cm] [Red ribbon: 5 cm 5 cm 5 cm 5 cm = 20 cm]
Try to incorporate scaling questions when teaching multiplication.

✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

\[ \square \times 5 = 20 \quad 3 \times \triangle = 18 \quad \square \times \bigcirc = 32 \]

✓ Partitioning NB. This is a big step so keep the numbers low to begin with. Children need to understand how to multiply by ten and one hundred before tackling larger numbers.

\[ 18 \times 5 = (10 \times 5) + (8 \times 5) \]
\[ = 50 + 40 \]
\[ = 90 \]

Show how to record this in a grid – split screen.

Children will continue to use arrays where appropriate leading into the grid method of multiplication.

\[ \begin{array}{c|c|c|c}
\times & 10 & & 4 \\
\hline
6 & & & \\
\hline
\end{array} \]

\[ (6 \times 10) + (6 \times 4) \]
\[ 60 + 24 \]
\[ 84 \]

Years 3 & 4

Grid method

TU x U - Short multiplication – multiplication by a single digit

Children need to understand what happens when we multiply by 10 and investigate what happens when we multiply by a multiple of ten

e.g. 20 x 8 can be calculated by multiplying 2 by 8 then making this ten times bigger or ‘putting back the zero’. We don’t say ‘adding 0’.

Page 30 of 41
Children will approximate first - 23 \times 8 \text{ is approximately } 25 \times 8 = 200

\[
\begin{array}{ccc}
\times & 20 & 3 \\
8 & 160 & 24 \\
\hline
& & 160 \\
+ & & 24 \\
\hline
& & 184 \\
\end{array}
\]

Children must know their times tables and have secure place value knowledge in order to do grid method multiplication.

**HTU x U - Short multiplication – multiplication by a single digit**

346 \times 9

Children will approximate first - 346 \times 9 \text{ is approximately } 350 \times 10 = 3500

Remind them of what happens when multiplying by a multiple of ten – you may ‘remove the zeroes’ for ease of calculation, but remember to ‘put back the zeroes’ e.g. 300 \times 9 = 3 \times 9 = 27, put back the two zeroes we took off from the original calculation = 2700

\[
\begin{array}{ccc}
\times & 300 & 40 & 6 \\
9 & 2700 & 360 & 54 \\
\hline
& & & 2700 \\
+ & & 360 & \\
\hline
& & & 3114 \\
\end{array}
\]

This method can also be used for multiplying ThHTU \times U (e.g. 4368 \times 7) and for multiplying decimal numbers by a single digit e.g. 4.9 \times 3. They should know that the decimal points line up under each other.

Children should estimate first – 4.9 \times 3 \text{ is approximately } 5 \times 3, \text{ which equals } 15.

\[
\begin{array}{ccc}
\times & 4 & 0.9 \\
3 & 12 & 2.7 \\
\hline
& & 12 \\
+ & 2.7 & \\
\hline
& & 14.7 \\
\end{array}
\]

Page 31 of 41
Y4 and beyond

Grid method for multiplying by a single digit will lead in to expanded short multiplication ‘fat sandwich’ and compact short multiplication ‘skinny sandwich’. Use a split screen – teach expanded alongside compact until children are secure enough to move over.

‘Fat sandwich’ shows how the grid method can be rearranged in to expanded short multiplication and then this can in turn be condensed to form the ‘skinny sandwich’ or compact method:

23 x 8 - from grid method to expanded and compact

<table>
<thead>
<tr>
<th>x</th>
<th>20</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>160</td>
<td>24</td>
</tr>
</tbody>
</table>

= 160

+24

= 184

23 x 8

= 184

It is crucial that children not only put a zero on the second row but that they also understand why – because we are multiplying by a tens number so the answer is ten times bigger.

346 x 9 – from grid method to expanded and compact

<table>
<thead>
<tr>
<th>x</th>
<th>300</th>
<th>40</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>2700</td>
<td>360</td>
<td>54</td>
</tr>
</tbody>
</table>

= 2700

+360

+54

= 3114

346 x 9

= 3114
TU x TU - Long multiplication – multiplication by more than a single digit.
Grid method to expanded long multiplication to compact.

e.g. 72 x 38 - Children will approximate first - 72 x 38 is approximately 70 x 40 = 2800

Using this method, children will also be able multiply HTU x TU and ThHTU x TU or x HTU etc.
They will also be able to multiply decimals with up to two decimal places by a single digit number and then 2-digit numbers. They should know that the decimal points line up under each other. It is essential children have a secure grasp of decimal place value and the multiplication methods before learning this.

E.g. 4.92 x 13. Children will approximate first - 4.92 x 13 is approximately 5 x 13 = 65

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>0.9</th>
<th>0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>40</td>
<td>9.0</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>2.7</td>
<td>0.06</td>
</tr>
</tbody>
</table>

+ 9
+ 0.2
+ 12
+ 2.7
+ 0.06

63.96

If at any point children are making significant errors, return to the previous stage to address misconceptions and secure understanding, and then transfer understanding to the more advanced method.

Children should have experience of being presented with a range of calculations/questions and needing to select and accurately use the most efficient method.

\[ + - + - + - + - + - + - + + \]
CALCULATION PROGRESSION FOR DIVISION

Mental Calculation Strategies for Division

These are a selection of mental calculation strategies:

- **Doubling and halving**
  Knowing that halving is dividing by 2

- **Deriving and recalling division facts**
  Year 1-2  2 x table, 10 x table, 5 x table
  Year 3  2 x table, 10 x table, 5 x table, 4 x table, 3 x table, 6 x table
  Year 4  Derive and recall division facts for all tables up to 12 x 12
  Year 5 & 6  Derive and recall quickly division facts for all tables up to 12 x 12

- **Using and applying division facts to scaled problems**
  Children should be able to utilise their tables knowledge to derive other facts.
  e.g. If I know 3 x 7 = 21, what else do I know?
  210 ÷ 3 = 70. 2100 ÷ 70 = 30. 210 ÷ 70 = 3. 21 ÷ 70 = 0.3

- **Dividing by 10 or 100**
  Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.
  Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

- **Use related facts**
  Given that 1.4 x 1.1 = 1.54
  What is 1.54 ÷ 1.4, or 1.54 ÷ 1.1?
Division – Written Calculation, Models & Images

Spend sufficient time – often longer than first thought – on teaching the inverse to increase children’s understanding, confidence and mental strategies. It is essential children are fluent with their number facts in order to readily access division. Division can be taught as repeated subtraction in the same way a multiplication is taught as repeated addition.
When teaching times tables include all division facts i.e all fact families.

Nursery & Reception
Children will begin to share items and count how many they have if objects are shared. They will start to understand the terms ‘groups’ and ‘share’.

Reception & Y1
Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s. Provide meaningful and concrete opportunities for children to divide.

Children will develop their understanding of division and use jottings to support calculation

✓ Sharing equally

6 sweets shared between 2 people, how many do they each get? ‘1 for you, 1 for me’

Y2
Children will continue to use cubes, other objects and jottings:

✓ Sharing equally

✓ Grouping or repeated subtraction
There are 6 sweets, how many people can have 2 sweets each?

Repeated subtraction using a number line

\[ 12 \div 3 = 4 \]

Using symbols to stand for unknown numbers to complete equations using inverse operations of times tables they know. When teaching times tables facts include the inverse.

\[ \square \div 2 = 4 \quad 20 \div \triangle = 4 \quad \square \div \triangle = 4 \]

Children will begin to relate dividing to finding fractions of shapes.

**Y3**

Ensure that the emphasis in Y3 is on grouping rather than sharing.

Children will continue to use:

Repeated subtraction using a number line

Children will use an empty number line to support their calculation.

\[ 24 \div 4 = 6 \]

Children should also move onto calculations involving remainders.

\[ 13 \div 4 = 3 \text{ r } 1 \]
✓ Using factor families, bar models and factor bugs to see the inverse relationship with multiplication for times tables they know well:

![Factor Families Diagram]

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

\[
26 \div 2 = \square \\
24 \div \triangle = 12 \\
\square \div 10 = 8
\]

**Y4**

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

\[
72 \div 5
\]

\[
0 \quad 2 \quad 7 \quad 12 \quad 17 \quad 22 \quad 27 \quad 32 \quad 37 \quad 42 \quad 47 \quad 52 \quad 57 \quad 62 \quad 67 \quad 72
\]

Moving onto:

![Repeated Subtraction Diagram]
Then onto the vertical method ‘chunking’:

**Short division TU ÷ U**

72 ÷ 3

\[
\begin{array}{c}
3 \left) \begin{array}{c}
72 \\
- 30 \\
\hline
42 \\
- 30 \\
\hline
12 \\
- 6 \\
\hline
6 \\
- 6 \\
\hline
0
\end{array}
\end{array}
\]

Answer : 24

98 ÷ 6

\[
\begin{array}{c}
6 \left) \begin{array}{c}
98 \\
- 60 \\
\hline
36 \\
- 36 \\
\hline
2
\end{array}
\end{array}
\]

Answer : 16 r 2

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to interpret the context of a question and decide whether to round their answer up or down accordingly. For example: 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
Answer: 8 (the remaining 6 apples still need to be placed into a box)

**Y5**

Children will continue to use written methods to solve short division TU ÷ U.

Children can start to subtract larger multiples of the divisor, e.g. 30x
Short division HTU ÷ U

196 ÷ 6

\[ \begin{array}{c|cc}
6 & 196 \\
- & 180 \\
\hline & 16 \\
\end{array} \]

Answer: 32 remainder 4 or 32 r 4

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

When children are secure in using the chunking method and understand how to find remainders accurately, they can be shown the compact formal written method:

196 ÷ 6

\[ \begin{array}{c|c|c}
6 & 19 & 16 \\
\hline & 32 & r 4 \\
\end{array} \]

As before, children need to be able to interpret the context of a question and decide whether to round their answer up or down accordingly. For example 240 ÷ 52 is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

Y6:

Formal method of long division HTU ÷ TU. Taking away multiples of the divisor ‘chunking’.

972 ÷ 36

\[ \begin{array}{c|c|c}
36 & 972 \\
- & 720 \\
\hline & 252 \\
\end{array} \]

Answer : 27
Formal method of short division HTU ÷ TU
Using this compact method is only suitable when dividing by numbers the children know multiples of and when children are secure with the method.

\[
\begin{array}{c}
11 \overline{) 496} \\
49 \\
\hline
56 \\
56 \\
\hline
0
\end{array}
\quad
\begin{array}{c}
5 \overline{) 931} \\
45 \times 2 \\
\hline
45 \\
93 \\
\hline
36
\end{array}
\]

\[
\begin{array}{c}
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\quad + \quad - \\
\end{array}
\]

Children should be taught to interpret the question and write remainders as integers, fractions or decimals depending on the context of the question i.e. if the children were dividing 32 by 10, the answer could be shown as \( 3 \frac{2}{10} \) which could then be written as \( 3 \frac{1}{5} \) in its lowest terms, or 3.2, or 3 remainder 2.

If at any point children are making significant errors, return to the previous stage to address misconceptions and secure understanding, and then transfer understanding to the more advanced method.

Children should have experience of being presented with a range of calculations/questions and needing to select and accurately use the most efficient method.

If a child is finding something challenging, is avoiding a task or is distracted, think:

HOW CAN WE MAKE THIS VISUAL OR USE CONCRETE RESOURCES TO SUPPORT?

HOW CAN WE MAKE THIS RELEVANT TO REAL LIFE?

HOW CAN WE REFER BACK TO WHAT THEY DO KNOW?

HOW CAN WE MAKE IT EASIER SO THE CHILD CAN SUCCEED?

HOW CAN WE MAKE THIS MORE FUN?

Written by Holly Kingham, October 2018

This policy has been written with reference to The National Curriculum, September 2013, the Lancashire Grid for Learning 'Progression through Calculations documents' and with the input of Fleet’s teachers.