Progression in Calculations
Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When teaching a new strategy it is important to start with numbers the child can easily manipulate so that they can understand the concept.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.
Progression in Teaching Addition

**Mental Skills**
- Recognise the size and position of numbers
- Count on in ones and tens
- Know number bonds to 10 and 20
- Add multiples of 10 to any number
- Partition and recombine numbers
- Bridge through 10

**Models and Images**
- Place value apparatus
- Place value cards
- Number tracks
- Numbered number lines
- Marked but unnumbered number lines
- Empty number lines
- Hundred square
- Counting stick
- Bead string
- Models and Images charts
- ITPs - Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens

**Key Vocabulary**
- add
- addition
- plus
- and
- count on
- more
- sum
- total
- altogether
- increase
Recognise numbers 0 to 10

1, 2, 3, 4, 5, 6
... there are 6 teddies

Count reliably up to 10 everyday objects

Find one more than a number

One more than three is four

Count in ones and tens

25 + 10 = 35

Begin to relate addition to combining two groups of objects

and makes 5

Begin to use the + and = signs to record mental calculations in a number sentence

6 + 4 = 10

Know doubles of numbers

3 + 2 = 5

Count along a number line to add numbers together

5 + 5 = 10
Know by heart all pairs of numbers with a total of 10 and 20

6 and how many more make 10?
6 + ___ = 10

10 = 5 + 5
10 = 1 + 9
10 = 2 + 8

1 + 2 = 3
2 + 1 = 3

Know that addition can be done in any order

2 count on 5
2 + 5 = 7

5 count on 2
5 + 2 = 7

2 + 5 = 7
5 + 2 = 7

3 + 5

Put the biggest number first and count on

8 + 7 = 15

Add two single-digit numbers that bridge 10

Begin to partition numbers in order to add

30p + 4p = 34p
Know which digit changes when adding 1s or 10s to any number

15 + 1 = 16
15 + 10 = 25
15 + 20 = 35

Adding two two-digit numbers (without bridging)
Counting in tens and ones
Partitioning and recombining

15 + 13 = 28

Adding two two-digit numbers (bridging through tens boundary)
Using a number line
OR
Using place value cards and Dienes to partition numbers and recombine

48 + 36 = 84

40 + 30 + 8 + 6
40 + 30 = 70
8 + 6 = 14
70 + 14 = 84
Standard written method

The previous stages reinforce what happens to the numbers when they are added together using more formal written methods.

Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

\[
\begin{align*}
48 &+ 36 \\
\hline \\
40 &+ 8 \\
30 &+ 6 \\
\hline \\
80 &+ 4 \\
\hline \\
10 &
\end{align*}
\]
Progression in Teaching Subtraction

Mental Skills

- Recognise the size and position of numbers
- Count back in ones and tens
- Know number facts for all numbers to 20
- Subtract multiples of 10 from any number
- Partition and recombine numbers (only partition the number to be subtracted)
- Bridge through 10

Models and Images

Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered lines
Hundred square
Empty number lines.
Counting stick
Bead strings

Models and Images Charts
ITPs - Number Facts, Counting on and back in ones and tens, Difference

Key Vocabulary

Subtract
Take away
Minus
Count back
Less
Fewer
Difference between
Begin to count backwards in familiar contexts such as number rhymes or stories.

Ten green bottles hanging on the wall ...

Five fat sausages frying in a pan ...

Continue the count back in ones from any given number.

Count backwards along a number line to 'take away'.

If I take away four shells there are six left.

Count back in tens.

Find one less than a number.

Begin to relate subtraction to 'taking away'.

Three teddies take away two teddies leaves one teddy.
Begin to use the - and = signs to record mental calculations in a number sentence.

\[ 6 - 4 = 2 \]

Maria had six sweets and she ate four. How many did she have left?

Know by heart subtraction facts for numbers up to 10 and 20.

\[
\begin{align*}
6 + ? &= 10 \\
10 - 6 &= ? \\
? + 6 &= 10 \\
10 - 4 &= 6
\end{align*}
\]

Subtract single digit numbers often bridging through 10.

\[ 15 - 7 = 8 \]

Begin to partition numbers in order to take away.

The difference between 11 and 14 is 3.

\[ 14 - 11 = 3 \]

\[ 11 + □ = 14 \]

Begin to find the difference by counting up from the smallest number.
Subtract 1 from a two-digit number

Subtract 10 from a two-digit number

Subtract multiples of 10 from any number

Partition the number to be subtracted (no exchanging)

Decide whether to count on or count back

Now where’s the answer?
Partitioning number to be subtracted - with exchanging (links to counting back on number line)

43 - 27 = 16

Expanded method
It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

<table>
<thead>
<tr>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

43 - 20 = 23
23 - 7 = 16

Standard written method
The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

\[ \begin{array}{c}
30 \\
40 \\
10 \\
+ 10 + 3 \\
- 20 + 7 \\
\hline
10 + 6 \\
\end{array} \]
Progression in Teaching Multiplication

Mental Skills

Recognise the size and position of numbers
Count on in different steps 2s, 5s, 10s
Double numbers to 10
Recognise multiplication as repeated addition
Quick recall of multiplication facts
Use known facts to derive associated facts
Multiplying by 10, 100, 1000 and understanding the effect
Multiplying by multiples of 10

Models and Images

Place value apparatus
Arrays
100 squares
Number tracks
Numbered number lines
Marked but unnumbered lines
Empty number lines.
Multiplication squares
Counting stick
Bead strings
Models and Images charts
ITPs - Multiplication grid, Number Dials, Multiplication Facts

Vocabulary

Lots of
Groups of
Times
Multiply
Multiplication
Multiple
Product
Once, twice, three times
Array, row, column
Double
Repeated addition

multiplication  product
once, twice, three times
double  groups of
repeated addition  lots of
array, row, column
multiply
times  multiple
Count in tens from zero

Count in twos from zero

Count in fives from zero

Know doubles and corresponding halves

Know multiplication tables to $10 \times 10$

Use known facts to work out new ones
Understand that ...

\[ 24 \times 20 = 24 \times 2 \times 10 \]
\[ 24 \times 50 = 24 \times 5 \times 10 \]

Understand multiplication as repeated addition

\[ 2 + 2 + 2 + 2 = 8 \]
\[ 4 \times 2 = 10 \]
2 multiplied by 4
4 lots of 2

Understand multiplication as an array

Understand how to represent arrays on a number line
Use place value apparatus to support the multiplication of U x TU

4 × 13

Use place value apparatus to support the multiplication of U x TU alongside the grid method

4 10 3

4

10 3

4 40 12

40 + 12 = 52

Use place value apparatus to represent the multiplication of U x TU alongside the grid method

4 × 13

10 10 3

4

10 10 3

4 40 40 12

80 + 12 = 92
Multiplying TU x TU

<table>
<thead>
<tr>
<th></th>
<th>30</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>300</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>12</td>
</tr>
</tbody>
</table>

= 330 +
= 132

462

56
× 27
1120 (56 × 20)
392 (56 × 7)
1512
1

Standard written method
Progression in Teaching Division

**Mental Skills**

Recognise the size and position of numbers  
Count back in different steps 2s, 5s, 10s  
Halve numbers to 20  
Recognise division as repeated subtraction  
Quick recall of division facts  
Use known facts to derive associated facts  
Divide by 10, 100, 1000 and understanding the effect  
Divide by multiples of 10

**Models and Images**

Counting apparatus  
Arrays  
100 squares  
Number tracks  
Numbered number lines  
Marked but unnumbered lines  
Empty number lines.  
Multiplication squares  
Models and Images charts  
ITPs - Multiplicationmainders  

**Vocabulary**

Lots of  
Groups of  
Share  
Group  
Divide  
Division  
Divided by  
Remainder  
Factor  
Quotient  
Divisible
Count back in tens

0 10 20 30

Count back in twos

? 2 4 6 8

Count back in fives

0 5 10 15

Know halves

Half of 6 is 3
½ of 6 = 3

Use known multiplication facts to work out corresponding division facts

If $2 \times 10 = 20$
then
$20 \div 10 = 2$
$20 \div 2 = 10$
Understand division as sharing

Understand division as grouping

How many 3s in 15?

12 divided into groups of 3 gives 4 groups
12 ÷ 3 = 4

Reinforce division as grouping through the use of arrays

12 divided into groups of 4 gives 3 groups
12 ÷ 4 = 3
Represent ‘groups’ for division on a number line using apparatus alongside the line.

18 divided into groups of 3

\[ 18 \div 3 = 6 \]

Understand division as repeated subtraction using a vertical line and apparatus to make the links.

\[ 18 \div 3 = 6 \]

\[ 18 - 3 (1 \times 3) \]

\[ 15 \]

\[ 18 \]

\[ 18 \div 3 = 6 \]

\[ 15 \]

\[ 12 \]

\[ 12 \]

\[ 9 \]

\[ 9 \]

\[ 6 \]

\[ 6 \]

\[ 3 \]

\[ 3 \]

\[ 0 \]

\[ 0 \]
Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the ‘chunk’.

\[
100 \div 7 = 14 \text{ r } 2
\]

\[
\begin{array}{c}
100 \\
-70 \quad (10 \times 7) \\
\hline
30 \\
-28 \quad (4 \times 7) \\
\hline
2
\end{array}
\]

\[
518 \div 7 = 74
\]

\[
\begin{array}{c}
518 \\
-350 \quad (50 \times 7) \\
\hline
168 \\
-140 \quad (20 \times 7) \\
\hline
28 \\
-28 \quad (4 \times 7) \\
\hline
0
\end{array}
\]

\[
518 \div 7 = 74
\]

\[
\begin{array}{c}
518 \\
-350 \quad (50 \times 7) \\
\hline
168 \\
-140 \quad (20 \times 7) \\
\hline
28 \\
-28 \quad (4 \times 7) \\
\hline
0
\end{array}
\]

\[
518 \div 7 = 74
\]

\[
\begin{array}{c}
518 \\
-350 \quad (50 \times 7) \\
\hline
168 \\
-140 \quad (20 \times 7) \\
\hline
28 \\
-28 \quad (4 \times 7) \\
\hline
0
\end{array}
\]

\[
518 \div 7 = 74
\]

\[
\begin{array}{c}
518 \\
-350 \quad (50 \times 7) \\
\hline
168 \\
-140 \quad (20 \times 7) \\
\hline
28 \\
-28 \quad (4 \times 7) \\
\hline
0
\end{array}
\]

\[
518 \div 7 = 74
\]

\[
\begin{array}{c}
518 \\
-350 \quad (50 \times 7) \\
\hline
168 \\
-140 \quad (20 \times 7) \\
\hline
28 \\
-28 \quad (4 \times 7) \\
\hline
0
\end{array}
\]

\[
518 \div 7 = 74
\]

\[
\begin{array}{c}
518 \\
-350 \quad (50 \times 7) \\
\hline
168 \\
-140 \quad (20 \times 7) \\
\hline
28 \\
-28 \quad (4 \times 7) \\
\hline
0
\end{array}
\]

Fact Box

- \(1 \times 7 = 7\)
- \(2 \times 7 = 14\)
- \(5 \times 7 = 35\)
- \(10 \times 7 = 70\)
- \(20 \times 7 = 140\)
- \(50 \times 7 = 350\)
- \(100 \times 7 = 700\)

Standard written method

Links directly to large chunk subtraction

\[
560 \div 24 = 23 \text{ r } 8
\]

\[
\begin{array}{c}
560 \\
-480 \quad (20 \times 7) \\
\hline
80 \\
-72 \\
\hline
8
\end{array}
\]

What facts do I know about the 7 times-table?