Taverham Sixth Form
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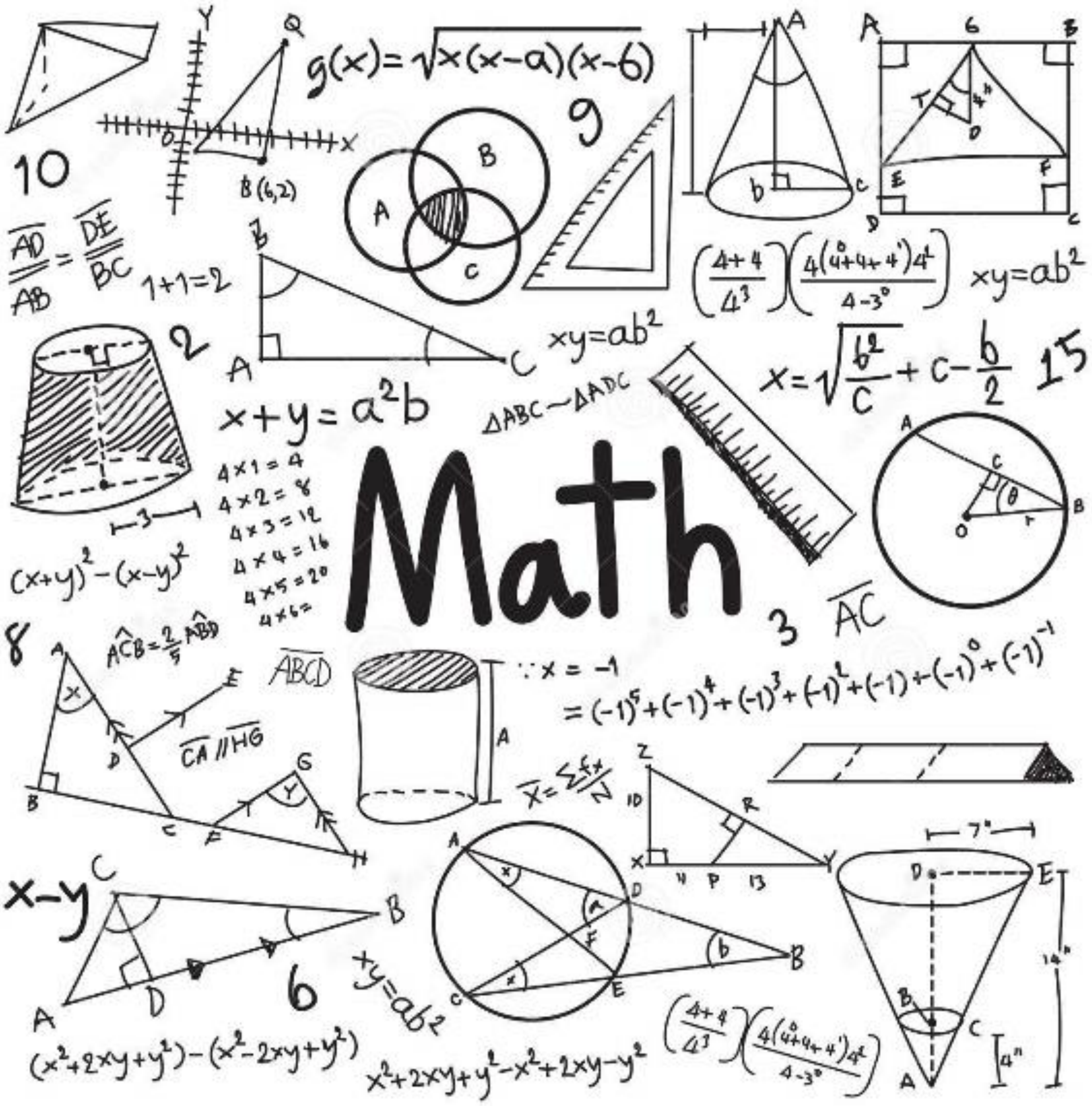
Essential Topics
A-Level Further Mathematics
Summer Work Booklet



Name ....

Indices

1)

(i) Write down the value of $\left(\frac{1}{4}\right)^0$. [1]

(ii) Find the value of $16^{-\frac{3}{2}}$. [3]

2)

Find the value of $\left(\dfrac{1}{2}\right)^{-5}$. [2]

3)

Find the value of $\left(\frac{1}{25}\right)^{-\frac{1}{2}}$. [2]

4)

Find the value of each of the following, giving each answer as an integer or fraction as appropriate.

(i) $25^{\frac{3}{2}}$ [2]

(ii) $\left(\dfrac{7}{3}\right)^{-2}$ [2]

5)

(i) Evaluate $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$. **[2]**

(ii) Simplify $\dfrac{(2ac^2)^3 \times 9a^2c}{36a^4c^{12}}$. **[3]**

6)

(i) Express $125\sqrt{5}$ in the form 5^k. **[2]**

(ii) Simplify $(4a^3b^5)^2$. **[2]**

Surds

1)

(i) Simplify $\sqrt{98} - \sqrt{50}$. [2]

(ii) Express $\frac{6\sqrt{5}}{2+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers. [3]

2)

(i) Express $\sqrt{48} + \sqrt{27}$ in the form $a\sqrt{3}$. **[2]**

(ii) Simplify $\frac{5\sqrt{2}}{3-\sqrt{2}}$. Give your answer in the form $\frac{b+c\sqrt{2}}{d}$. **[3]**

3)

(i) Express $\frac{1}{5+\sqrt{3}}$ in the form $\frac{a+b\sqrt{3}}{c}$, where a, b and c are integers. [2]

(ii) Expand and simplify $(3 - 2\sqrt{7})^2$. [3]

4)

You are given that $a = \frac{3}{2}$, $b = \frac{9-\sqrt{17}}{4}$ and $c = \frac{9+\sqrt{17}}{4}$. Show that $a + b + c = abc$. [4]

Proof

1)

n is a positive integer. Show that $n^2 + n$ is always even. [2]

2)

Prove that, when n is an integer, $n^3 - n$ is always even. **[3]**

3)

(i) Prove that 12 is a factor of $3n^2 + 6n$ for all even positive integers n. [3]

(ii) Determine whether 12 is a factor of $3n^2 + 6n$ for all positive integers n. [2]

4)

Factorise $n^3 + 3n^2 + 2n$. Hence prove that, when n is a positive integer, $n^3 + 3n^2 + 2n$ is always divisible by 6. **[3]**

Solving linear inequalities

1)

Solve the inequality $\frac{5x-3}{2} < x+5$. **[3]**

2)

Solve the inequality $\frac{3(2x+1)}{4} > -6$. [4]

Solving equations

1)

Solve the equation $\frac{4x+5}{2x} = -3$. [3]

2)

Solve the equation $\frac{3x+1}{2x} = 4$. **[3]**

3)

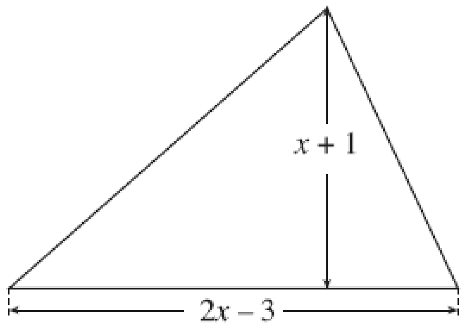
Solve the equation $y^2 - 7y + 12 = 0$.

Hence solve the equation $x^4 - 7x^2 + 12 = 0$. [4]

Forming and solving equations

1)

The triangle shown in Fig. 10 has height $(x+1)$ cm and base $(2x-3)$ cm. Its area is 9 cm^2.



Not to scale

Fig. 10

(i) Show that $2x^2 - x - 21 = 0$. [2]

(ii) By factorising, solve the equation $2x^2 - x - 21 = 0$. Hence find the height and base of the triangle. [3]

2)

Fig. 9 shows a trapezium ABCD, with the lengths in centimetres of three of its sides.

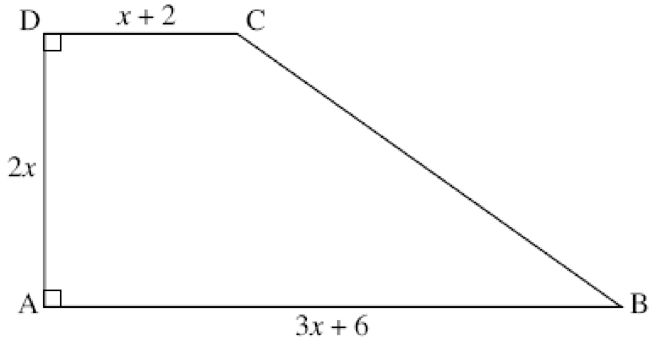


Fig. 9

This trapezium has area $140\,\text{cm}^2$.

(i) Show that $x^2 + 2x - 35 = 0$. **[2]**

(ii) Hence find the length of side AB of the trapezium. **[3]**

Completing the square and turning points

1)

(i) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$. **[3]**

(ii) Write down the coordinates of the minimum point on the graph of $y = x^2 + 6x + 5$. **[2]**

2)

(i) Write $x^2 - 7x + 6$ in the form $(x - a)^2 + b$. [3]

(ii) State the coordinates of the minimum point on the graph of $y = x^2 - 7x + 6$. [2]

(iii) Find the coordinates of the points where the graph of $y = x^2 - 7x + 6$ crosses the axes and sketch the graph. [5]

3)

(i) Write $3x^2 + 6x + 10$ in the form $a(x + b)^2 + c$. **[4]**

(ii) Hence or otherwise, show that the graph of $y = 3x^2 + 6x + 10$ is always above the x-axis. **[2]**

Discriminant and roots

1)

Find the discriminant of $3x^2 + 5x + 2$. Hence state the number of distinct real roots of the equation $3x^2 + 5x + 2 = 0$. **[3]**

2)

Find the set of values of k for which the equation $2x^2 + 3x - k = 0$ has no real roots. [3]

3)

Find the set of values of k for which the equation $2x^2 + kx + 2 = 0$ has no real roots. [4]

4)

Prove that the line $y = 3x - 10$ does not intersect the curve $y = x^2 - 5x + 7$. **[5]**

Changing the subject of a formula

1)

Make y the subject of the formula $a = \frac{\sqrt{y} - 5}{c}$. **[3]**

2)

Rearrange the formula $c = \sqrt{\frac{a + b}{2}}$ to make a the subject. [3]

3)

The volume V of a cone with base radius r and slant height l is given by the formula

$$V = \tfrac{1}{3}\pi r^2 \sqrt{l^2 - r^2}.$$

Rearrange this formula to make l the subject. **[4]**

4)

Make a the subject of the equation

$$2a + 5c = af + 7c.$$ [3]

5)

Make x the subject of the formula $y = \frac{1 - 2x}{x + 3}$. **[4]**

Equation of a straight line

1)

A line has equation $3x + 2y = 6$. Find the equation of the line parallel to this which passes through the point $(2, 10)$. [3]

2)

(i) Find the equation of the line passing through A $(-1, 1)$ and B $(3, 9)$. [3]

(ii) Show that the equation of the perpendicular bisector of AB is $2y + x = 11$. [4]

Intersection of two lines

1)

Solve the simultaneous equations $y = x^2 - 6x + 2$ and $y = 2x - 14$. Hence show that the line $y = 2x - 14$ is a tangent to the curve $y = x^2 - 6x + 2$. [5]

2)

Find algebraically the coordinates of the points of intersection of the curve $y = 4x^2 + 24x + 31$ and the line $x + y = 10$. **[5]**

3)

Find the coordinates of the points of intersection of the circle $x^2 + y^2 = 25$ and the line $y = 3x$. Give your answers in surd form. [5]

4)

A circle has equation $x^2 + y^2 = 45$.

(i) State the centre and radius of this circle. [2]

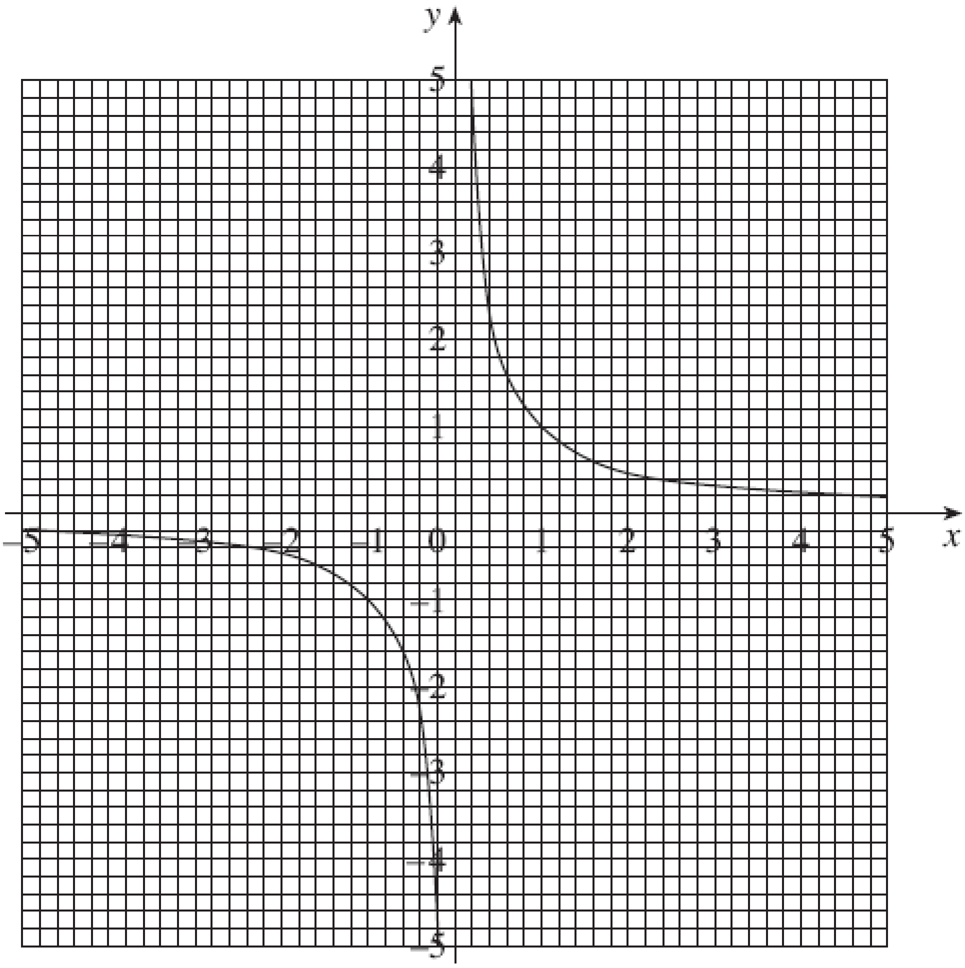
(ii) The circle intersects the line with equation $x + y = 3$ at two points, A and B. Find algebraically the coordinates of A and B.

Show that the distance AB is $\sqrt{162}$. [8]

Using graphs to solve equations

1)

The insert shows the graph of $y = \frac{1}{x}$, $x \neq 0$.



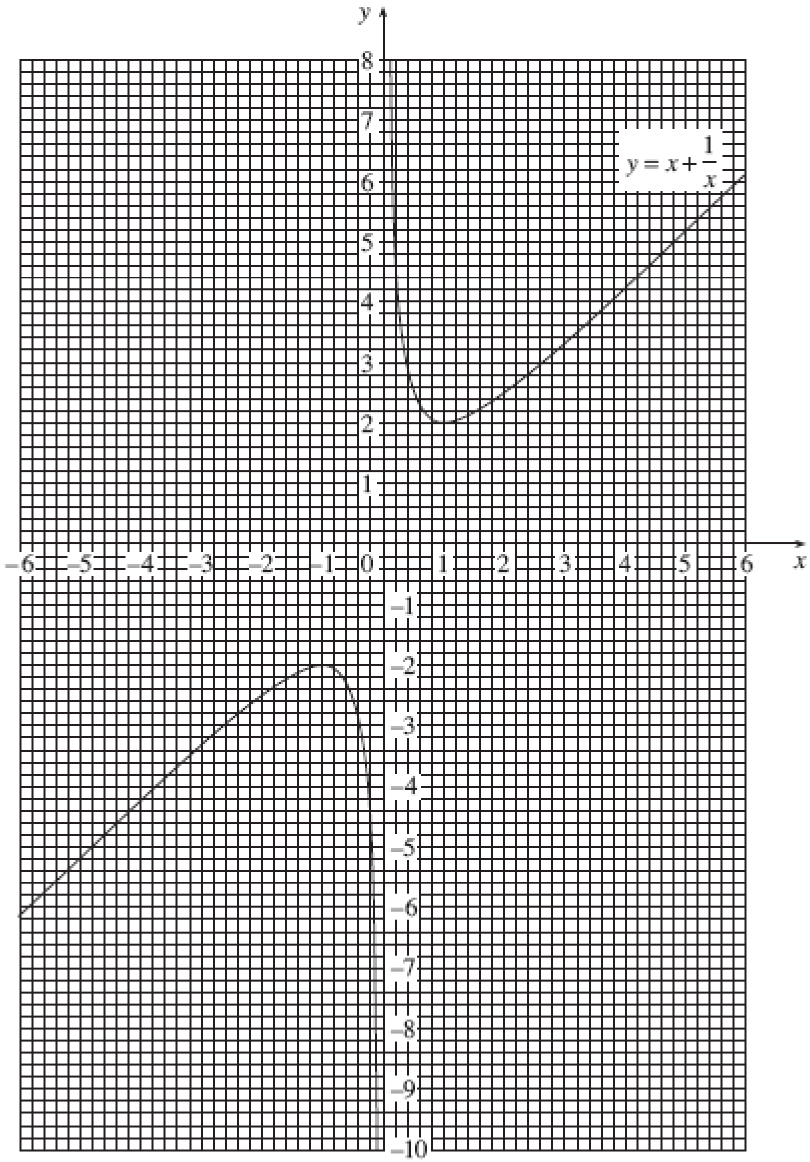
(i) Use the graph to find approximate roots of the equation $\frac{1}{x} = 2x + 3$, showing your method clearly. [3]

(ii) Rearrange the equation $\frac{1}{x} = 2x + 3$ to form a quadratic equation. Solve the resulting equation, leaving your answers in the form $\frac{p \pm \sqrt{q}}{r}$. [5]

(iii) Draw the graph of $y = \frac{1}{x} + 2$, $x \neq 0$, on the grid used for part (**i**). [2]

2)

The graph of $y = x + \frac{1}{x}$ is shown on the insert. The lowest point on one branch is $(1, 2)$. The highest point on the other branch is $(-1, -2)$.



Use the graph to solve the following equations, showing your method clearly.

(A) $x + \frac{1}{x} = 4$ [2]

(B) $2x + \frac{1}{x} = 4$ [4]

Transformation of graphs

1)

The point P (5, 4) is on the curve $y = f(x)$. State the coordinates of the image of P when the graph of $y = f(x)$ is transformed to the graph of

(i) $y = f(x - 5)$, **[2]**

(ii) $y = f(x) + 7$. **[2]**

2)

The curve $y = f(x)$ has a minimum point at (3, 5).

State the coordinates of the corresponding minimum point on the graph of

(i) $y = 3f(x)$, **[2]**

(ii) $y = f(2x)$. **[2]**

3)

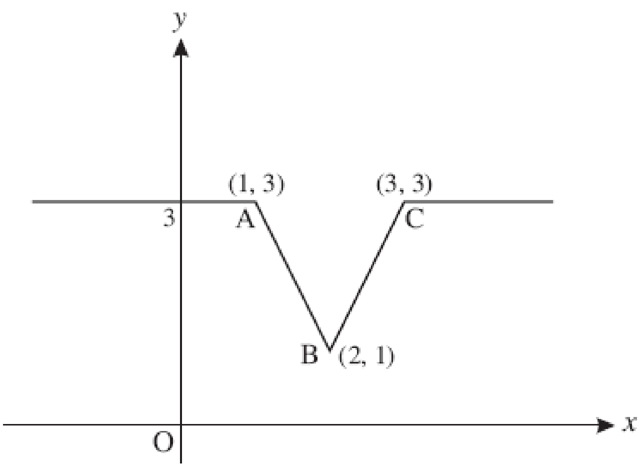


Fig. 4

Fig. 4 shows a sketch of the graph of $y = f(x)$. On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to A, B and C.

(i) $y = 2f(x)$ [2]

(ii) $y = f(x + 3)$ [2]

4)

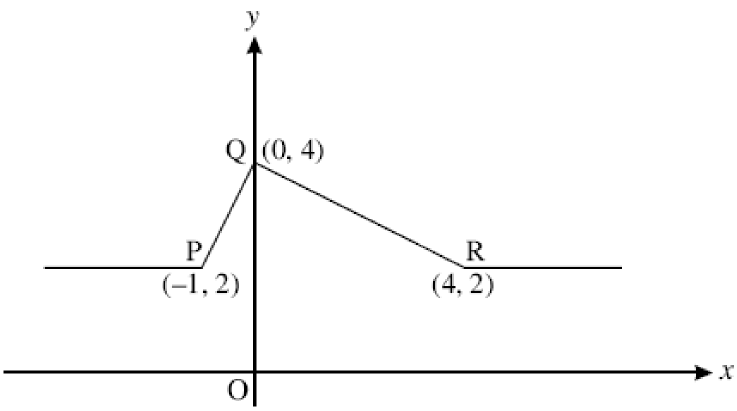


Fig. 5

Fig. 5 shows a sketch of the graph of $y = \mathrm{f}(x)$. On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to P, Q and R.

(i) $y = \mathrm{f}(2x)$ **[2]**

(ii) $y = \frac{1}{4}\mathrm{f}(x)$ **[2]**

5)

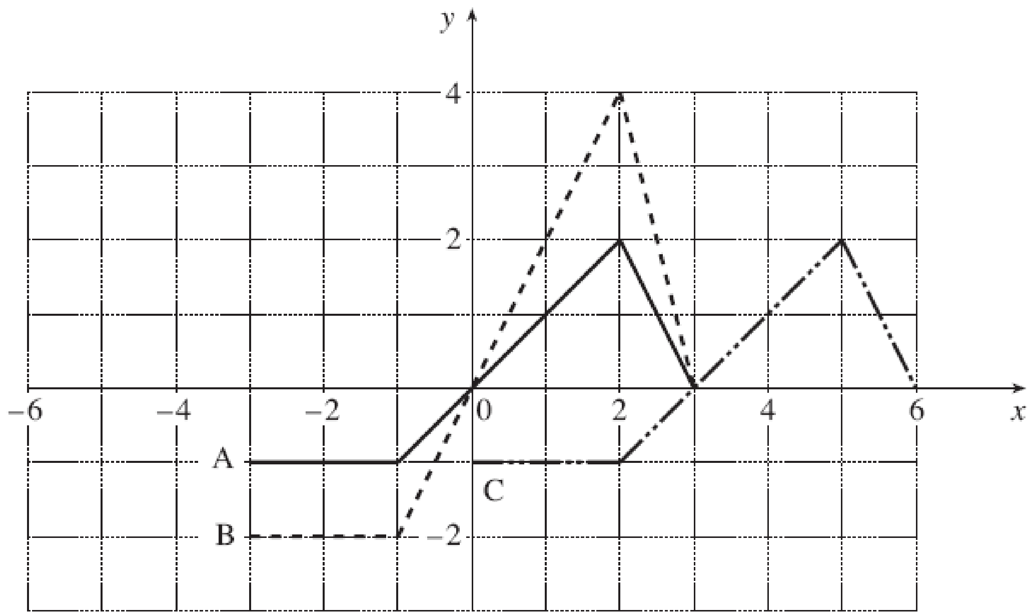


Fig. 3

Fig. 3 shows sketches of three graphs, A, B and C. The equation of graph A is $y = \mathrm{f}(x)$.

State the equation of

(i) graph B, [2]

(ii) graph C. [2]

6)

The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Write down an equation for the translated curve. You need not simplify your answer. [2]

7)

Describe fully the transformation which maps the curve $y = x^2$ onto the curve $y = (x + 4)^2$. **[2]**

Trigonometric graphs

1)

Sketch the curve $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

Solve the equation $\sin x = -0.68$ for $0° \leqslant x \leqslant 360°$. [4]

2)

Sketch the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$.

On the same axes, sketch the graph of $y = \cos 2x$ for $0° \leqslant x \leqslant 360°$. Label each graph clearly. [3]