Mastering Mathematics

17.01.18
Mastery and the National Curriculum

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have **conceptual understanding** and are able to recall and apply their knowledge rapidly and accurately to problems

- **reason** mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

- can **solve problems** by applying their mathematics to a variety of **routine and non-routine problems with increasing sophistication**, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Children must master the curriculum for their year group, so that they have firm foundations to build on the following year.
Our position in the world in 2015 – 10th

 Apparently, now, with other countries joining the survey – 28th
Our top performing students perform equally with Shanghai students, but...
What is mastery?

If you drive a car, imagine the process you went through...

- The very first drive, lacking the knowledge of what to do to get moving
- The practice, gaining confidence that you are able to drive
- The driving test, fairly competent but maybe not fully confident
- A few years on, it’s automatic, you don’t have to think about how to change gears or use the brake
- Later still, you could teach someone else how to drive or drive in any situation
What is mastery?

Mastery in mathematics is similar. It involves:

• Deep and sustainable learning
• Ability to build on something already mastered
• Ability to reason about a concept and make connections to other concepts
• Procedural and conceptual fluency (can’t solve problems without these)
• The understanding of how and why it all works

Mastery is a continuum... mastery at a particular point of time that is sufficient mastery for that stage of learning and then built on at a later stage.
The wise men of mathematics

Concrete – Pictorial – Abstract.

**Jerome Bruner.** American. 1915 – 2016

Variation Theory. **Zoltan Dienes.** Hungarian. (1916 – 2014)

Conceptual and procedural understanding.

Using what we already know

**David Ausubel:** American - 1918 - 2008

Origins of thinking

**Jean Piaget:** Swiss – 1896 – 1980

Multiple intelligence

**Martin Gardner:** American: 1914 - 2010

Social theory

**Lev Vygotsky:** Russian – 1896 - 1934

Practical apparatus

**Georges Cuisenaire:** Belgium: 1891 - 1975
Mastery is nothing new!

Bruner emphasized that ideas are not merely repeated but…”revisited later with greater precision and power until the students achieve the reward of mastery” Bruner, 1979.

Benjamin Bloom (remember him!) said in 1971, that the only way to reduce the gap between the lowest and highest attainers is to develop a mastery curriculum.
Teaching for Mastery

- Access
- Pattern
- Making Connections

Representation & Structure
- Coherence

Mathematical Thinking
- Chains of Reasoning
- Making Connections

Small steps are easier to take

Variation
- Procedural
- Conceptual
- Making Connections

Fluency
- Number Facts
- Table Facts
- Making Connections
Teaching for mastery

Dimensions of Depth

1. Factual knowledge
2. Conceptual Understanding
3. Procedural fluency
4. Language and communication
5. Thinking mathematically

Drury, 2014 (2,4,5)
Teaching for mastery

Dimensions of Depth

1. Factual knowledge
   Counting (e.g. knowing 4 comes after 3 and before 5)
   Number bonds for all numbers to 10
   Counting in steps of different sizes
   Doubling and halving of numbers to 10
   Multiplication facts
Factual knowledge
### Factual knowledge

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### Use Cases

- **If I know my 2s, I know my 4s and 8s**
- **If I know my 3s, I know my 6s and 12s**
- **I can use my 2s and 5s to work out my 7s**
- **I can use my 4s and 5s to work out my 9s**
### Multiplication and division facts

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### Factual knowledge

- **Multiplication and division facts**: These are basic mathematical operations used in everyday calculations. Understanding these facts is crucial for more complex mathematical concepts. For example, knowing that $2 \times 2 = 4$ helps in calculating areas of squares, while $9 \times 3 = 27$ is useful in understanding growth rates or in budgeting. Each product represents a specific numerical relationship that can be applied in various contexts such as cooking recipes, construction measurements, or financial planning.
Factual knowledge

Multiplication and division facts

<table>
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<th>6 x 6 = 36</th>
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Pineapples: 9 x 9 = 81

Lemons: 10 x 9 = 90

Watermelons: 11 x 9 = 99

Oranges: 12 x 9 = 108
### Factual knowledge

### Multiplication and division facts

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<th>10 x 10 = 100</th>
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Where mastery begins……

Key performance indicators for Year R

Counting:

- Stable order principle (number names)
- One to one principle (one number for each item)
- Cardinal principle (last number)
- Order irrelevance principle (conservation)
- Abstraction principle

Small intervention groups. 5 minutes a day to ensure progress through these steps.

Along with this:
Knowing all about numbers to 10
Ability to subitise
Number recognition

Have all of your children achieved these principles?

Gelman and Gallistel
Show me 4 in as many ways as you can.
What do you know about 4?
Odd or even?
What is it greater than? What is it less than?
Can you count out 4 from a larger group?
Can you show the numeral?
Number pairs for 4: part, part, whole model leading to number facts and generalisation.
Play Your Cards Right
Counting

- Counting is the starting point of number work. It forms an essential part of children’s developing understanding of numbers.

- However it is not the best foundation for calculating. Children need to learn to manipulate numbers in particular ways.

- To calculate they need to move on from counting.

- Children who are still counting using their fingers in Year 2 will struggle with mathematics in KS2.
Three key aspects of number sense

Counting

Composition

Comparing
Three key aspects of number sense

Counting:
Knowing the number names in order, forwards and backwards.
Understanding how to count objects, events or actions in ones and also in twos, fives and tens.

Counting 16 beads, 5 claps, 10 stairs
Count down as 5 buns are eaten
Count up money in 10p pieces
Three key aspects of number sense

Comparing:
- Having a feel for the relative sizes of numbers
- Putting numbers in order
- Estimating

Know that 6 is smaller than 8 and bigger than 2
- Being able to see that a group of objects contains about 10
- Ordering 10, 4, 9, 2 from largest to smallest
Three key aspects of number sense

Composition:
Understanding how each number can be made up in different ways by adding and subtracting
Knowing how our number system uses groups of hundreds, tens and ones

Recognising that $6 = 2 + 4$ or $7 - 1$
Understand that 15 can be made from one ten and 5 ones, 15 ones
Number of the week

Some children enjoy and are good at counting. Others have less experience. All will do well if we provide a rich environment of situations and problems that the child finds engaging. Fingers hugely important. How else can you show 6..how else...how else?
Build it
Draw it
Talk it

5 + 1 = 6
4 + 2 = 6
3 + 3 = 6
2 + 4 = 6
1 + 5 = 6
**Numicon**

Show me 1
How do you know it is 1?

Show me 5
How do you know it is 5?
What can you tell me about 5? What else? What else?

Show me 8
How do you know it is 8?
What can you tell me about 8? What else? What else?
Numicon

Show me 10
How do you know it is 10?
What can you tell me about 10?
What else? What else?

Show me 11
How do you know it is 11?
What can you tell me about 11?
What else? What else?

Make a line of Numicon from 1 to 20
How do you know that you are correct?
Do you play games with your children?
• Games can develop reasoning skills
• They can help promote social skills
• They are fun!
Teaching for mastery

Dimensions of Depth

2. Conceptual understanding
   Understanding equivalence
   Understanding place value
   Understanding what addition is
   Understanding what subtraction is
   Understanding of the relationship between addition and subtraction
   Understanding of commutativity
   Understanding of inverse
   As above for multiplication and division
Understanding equivalence

the same as

equivalent

equal

balance

Not the answer to a calculation!
Equivalence

10 = 6 + ___
? + 8 = 14
13 - ? = 9
11 + 7 = 4 + ?
Understanding place value

- Positional
- Multiplicative
- Additive
- Base 10

(Ross 1989)
Do these represent an understanding of place value?

JULIA ANGHIleri1, MEINDERT BEISHUIZEN2 and KEES VAN PUTTEN (2001)
Developing place value:
teens numbers
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Teaching for mastery

Dimensions of Depth

3. Procedural fluency
   Understanding the processes of addition and subtraction
   Understanding that subtraction can check an addition and addition can be used to check a subtraction
   As above for multiplication and division
Begin teaching addition and subtraction together so children can see the relationship between them. Then...

Focus on addition with subtraction as a check.
Focus on subtraction with addition as a check.

Use a calculator!!
Addition structures

Augmentation
Augend add addend equals sum

Aggregation
Addend add addend equals sum
Models for addition

Aggregation: combining two amounts

Augmentation: adding on to an amount

Both rely on counting, which allows for errors

$6 + 9$
Models for addition

Numicon or similar manipulatives:
enables addition without continual counting

These patterns of visual representations of numbers show that 6 + 9 is the same as 10 + 5, which is 15 – requires familiarity of resources used.
Towards the written method

Can you spot a pattern?

Make up a story.

Build it
Draw it
Speak it
Write it!
Towards the written method

25 + 34 = 59

35 + 34 = 69

45 + 34 = 79

55 + 34 = 89
Manipulatives for addition

45 + 77

What’s the same?
What’s different?

45
+ 77
110
12
122

Build it
Draw it
Speak it
Write it!
The Exchange Game

• Throw a dice, collect that number of ones
• Repeat this
• When you have 10, exchange for a 10s counter or rod
• The first player to reach 20 is the winner
  ...or 30
  ...or 40
  ...or 50
Structures for subtraction

Reduction:
Removing items from a set
Minuend subtract subtrahend equals difference

15 - 9
Structures for subtraction

Comparison or difference
Minuend subtract subtrahend equals difference

15 - 9
Resources for subtraction

Numicon or similar manipulatives:
enables subtraction without continual counting

Build it
Draw it
Speak it
Write it!

These patterns of visual representations of numbers show that 9 is 6 less than 15
Towards the written method

Can you spot a pattern?

Make up a story.
Towards the written method

Build it
Draw it
Speak it
Write it!

87 - 42
87 - 52
87 - 62
87 - 72
87 - 82
Manipulatives for subtraction

82 - 47

Build it
Draw it
Speak it
Write it!
Begin teaching multiplication and division together so children can see the relationship between them. Then...

Focus on multiplication with division as a check.
Focus on division with multiplication as a check.
Structures for multiplication

• Ratio (scaling)

• Repeated addition (grouping)

Multiplicand multiplied by multiplier equals product
Structures for division

- Ratio (scaling)
- Repeated subtraction (grouping)
- Sharing

Dividend divided by divisor equals quotient
Doubling: link to repeated addition and grouping

First steps for multiplication

What happens if we add one group of a number to another group of that number?

Make up a story.
Halving: link to repeated subtraction and grouping

First steps for division

Can we take one group of a number away and be left with another group of that number?

Build it
Draw it
Speak it
Write it!

Make up a story.
I can see 1, 2, 3, 4…12

I can see 3 rows of 4

If you go round the other side, it’s 4 rows of 3

I can see 3 and 3 and 3 and 3

I can see 4 and 4 and 4

It’s 4 trebled!
Scaling up

Twice as big

3 times as big

4 times as big

This is the beginning of ratio
Half the size

$\frac{1}{3}$ of the size

$\frac{1}{4}$ of the size
Scaling is natural!!
We can all do it.
The bar model...

• It is a mathematical representation of a word problem

• It is a representation that reveals the structure of a word problem

• A way of ‘acting out a problem’

• It is not a calculating tool
Why use the bar model?

Sam had 5 times as many marbles as Tom. If Sam gives 26 marbles to Tom, the two friends will have exactly the same amount. How many marbles do they have altogether?
To become proficient at using this model, children need to be introduced to it early in their education...

There are 3 footballs in the red basket and 2 footballs in the blue basket.
How many footballs are there altogether?

3 Balls 2 Balls
Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?
Drinks time!
How many drinks are there?

How many more do we need?
Theo has 3 cars and 2 lorries. How many toys has he got?

Theo has got ________ toys.
Addition and subtraction

This bar model can then help the children solve, for example, missing number problems:

Freddy scored 59 points on the computer game.
Samir scored 34 points more.
How many points did Samir score?

Jenny had a collection of shells.
She gave her friend 23 of them.
She was left with 46.
How many shells did she have before giving some to her friend?

Sam went for a run.
He ran 12km to the shop and 5km back.
Then he stopped to talk to a friend.
How much further did he need to run to get home?
Two apples and a mango cost £2.
Two apples and three mangoes cost £4.
Find the cost of a mango.
The difference between two numbers is 27. If the larger number is 4 times the smaller number, find the sum of the two numbers.
Peter had 5 marbles.
Julie had 3 times as many.
How many marbles did they have altogether

What if Peter had 15 marbles?
What if Peter had 25 marbles?
What if Peter had 50 marbles?
What if Peter had 75 marbles?

**Variation:**
Begin by exploring the idea of the problem... Peter and Julie had some marbles. How many marbles could they have had? What if Julie had twice as many marbles as Peter? What if Peter had half the number of Julie’s marbles? What questions can you ask?
Lauren has some cherries.
She eats 2 of them.
Then she eats half of what is left.

She now has 6 cherries.

How many did she start with?
Fractions

Ella has some cherries. She eats one half of them and has 4 left. How many did she have to begin with?

Ella has some cherries. She eats one half of them and has 6 left? How many did she have to begin with?

Ella has some cherries. She eats one quarter of them and has 6 left? How many did she have to begin with?

Ella has some cherries. She eats one third of them and has 6 left? How many did she have to begin with?
Fractions

\[ \frac{1}{2} \text{ of the sweets in the tin were chocolates.} \]

\[ \frac{1}{4} \] were toffees.

The rest were strawberry creams.

There were 5 strawberry creams.

How many sweets were in the tin?

What if there were 8 strawberry creams?

What if there were 12 strawberry creams?

What if there were 25 strawberry creams?
KS2 2012

In a class, 18 of the children are girls.

A quarter of the children in the class are boys.

Altogether, how many children are there in the class?

Show your working

With the bar model embedded, Year 3, possibly Year 2, could easily solve this!
Show me....

If my whole is....

If my parts are....
In a Year 3, \(\frac{1}{4}\) of the children are boys.
There are 30 girls.
How many children are in Year 3?

What if there were 24 girls?
What if there were 33 girls?
What if there were 36 girls?
What if there were 48 girls?
In Year 3, \( \frac{3}{4} \) of the children were boys.
There are 30 girls.
How many children are in Year 3?

What if there were 24 girls?
What if there were 33 girls?
What if there were 36 girls?
What if there were 48 girls?
In Year 3, \( \frac{2}{5} \) of children are boys.
There are 30 girls.
How many children are in Year 3?

What if there were 24 girls?
What if there were 33 girls?
What if there were 36 girls?
What if there were 48 girls?

Make up some other numbers of girls that would work with this problem!
A gardener plants tulip bulbs in a flower bed. She plants 3 red bulbs for every 4 white bulbs. With the bar model embedded, Year 2 could do this!

How many **white** bulbs does she plant?

What if she planted 90 red bulbs? What if she planted 300 red bulbs?
Two numbers are in the ratio $4 : 5$

One of the numbers is $60$

There are two possible values for the other number.

What are the two possible values?
Manipulatives and visual representations are vital because they:

- Help children to make sense of arithmetic
- Help teachers see what children understand
- Increase children’s engagement and enjoyment
- Develop visual images and understanding
- Help children to work together and share ideas
- Are tools to help children solve problems, investigate patterns and relationships, demonstrate results and reasoning
- Provide a bridge to abstract thinking